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Introduction to weakly b- transitive maps on topological spaces

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Abstract: The concept of b-regular open set was introduced by N. Rajesh.. The aim of this paper is to introduce and characterize weakly b-transitive maps by using b-regular open sets and investigate some of its properties. Further, we introduce the notions of weakly b- minimal mapping. We have proved that every b-transitive map is a weakly b- transitive map but the converse not necessarily true, and that every b-minimal map is a weakly b- minimal map, but the converse not necessarily true.

Keyword: Weakly b- Transitive Maps, Weakly b- Minimal Functions, b- Continuous, Weakly b- Irresolute

1. Introduction

Functions and of course irresolute functions stand among the most important notions in the whole of mathematical science. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In 1972, Crossley and Hildebrand [1] introduced the notion of irresoluteness. . Many different forms of irresolute functions have been introduced over the years. The concept of b-regular open (resp. b-open) sets was introduced by N. Rajesh [2] (resp. Andrijevic [3]) . This type of sets was discussed by Ekici and Caldas [4] under the name of -open sets. The class of b-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. A subset A of (X, τ) is said to be b-open [2] (or γ-open [4]) (resp. α-open [5]) if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$ (resp. $A \subseteq Int(Cl(Int(A)))$). A set A is said to be b-regular open [6] The complement of b-open set is called b-closed. The concept of weakly b-irresolute functions was introduced and investigated by N. Rajesh[3] . In this paper, I will introduce and characterize weakly b-transitive maps and I will continue the study of related functions with b-regular open [6] sets .The family of all b-open (resp. α-open, b-closed, b-regular open) subsets of (X, τ) will be denoted by BO (X) (resp. τ^{α} or αO(X), BC(X), BR(X)). BO(X, x)={V ∈ BO(X)/ x ∈ V}. In this paper, I will define a new class of topological b-transitive maps called weakly b- transitive maps and a new class of weakly b- minimal maps. I have shown that *every b-transitive map is a weakly b-transitive map, but the converse not necessarily true and that every b-minimal map is a weakly b--minimal map, but the converse not necessarily true* we will also study some of their properties. Throughout this paper, the word "space "will mean topological space.

2. Transitivity and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a system (X, f) short for a topological dynamical system (X, f) [7] we mean a topological space X together with a continuous map $f : X \to X$. The space X is sometimes called the phase space of the system (X, f). A subset A *of* X is called f -*invariant* if $f(A) \subseteq A$.

In this section, I generalize topologically transitive maps to weakly b- transitive maps that may not be b-transitive. I will define the weakly b- transitive maps on a space (X, τ) and weakly b- minimal maps that may not be b-minimal,), and I study some of their properties and prove some results associated with these new definitions I investigate some properties and characterizations of such maps.

Definition 2.1.

A system (X, f) is called weakly b-*minimal* if X does not contain any non-empty, proper, b-regular f -*invariant* subset of X. In such a case, the map f itself is called weakly b-minimal.

Given a point x in a system (X, f), $O_f(x) = \{x, f(x), f^2(x), ...\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω -limit set, i.e. the set of limit points of the sequence $x, f(x), f^2(x), ...$ we will study the Existence of minimal sets. Given a dynamical system (X, f), a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f).

The basic fact discovered by G. D. Birkhoff, this fact says that in any compact system (X, f) there are minimal sets. Since any orbit closure is invariant, we get that *any compact orbit closure contains a minimal set*. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) either are disjoint or coincide. A minimal set A is strongly *f-invariant,* i.e. *f(A) is a subset of A*. provided it is compact Hausdorff space (i.e. T_2-space). For more knowledge about minimal systems, topologically transitive maps and minimal sets. see references [9], [10], [11] and [12].

Definition 2.2

A subset A of a topological space (X,τ) is said to be nowhere b-dense, if its b-closure has an empty b-interior, that is, $\operatorname{int}_b(Cl_b(A)) = \varphi$.

Definition 2.3

[3] A function $f : X \to X$ is called b-irresolute if the inverse image of each b-open set is a b-open set in X.

Definition 2.4[3]

Let (X, τ) be a topological space, $f : X \to X$ is said to be weakly b-irresolute map if for each $x \in X$ and each $V \in BO(X, f(x))$, there exists $U \in BO(X, x)$, such that $f(U) \subset bCl(V)$.

Clearly, every b-irresolute map is weakly b-irresolute but the converse is not true, unless, the space X is b-regular space, via the following theorem:

Theorem 2.5 [3]

Let Y be b-regular space. Then a function $f : X \to Y$ is weakly b-irresolute if and only if it is b-irresolute.

I will define some new definitions as follows:

Definition 2.6.

A system (X, f) is called *topologically weakly b-mixing if* for any pair U, V of *non-empty* b-regular open sets there exists $N \in \mathrm{N}$ such that for all n ≥ N we have $f^n(U) \cap V \neq \varphi$.

Topologically weakly b- mixing conveys the idea that each b-regular open set U, after iterations of f, for each b-regular open set V, for all n sufficiently large, $f^n(U)$ intersects V

Definition 2.7

Let (X, τ) be a topological space, and $f : X \to X$ a continuous map, then f is said to be a topologically transitive map if for every pair of open sets U and Vin X there is a positive integer n such that $f^n(U) \cap V \neq \varphi$.

Definition 2.8.

A function $f : X \to X$ is called weakly br-homeomorphism if f is weakly b-irresolute bijective and $f^{-1} : X \to X$ is weakly b-irresolute.

Definition 2.9

Two topological dynamical systems $f : X \to X$, $x_{n+1} = f(x_n)$ and $g : Y \to Y$, $y_{n+1} = g(y_n)$ are said to be topologically weakly br-conjugate if there is weakly $br-$homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$ *(i.e. h(f(x)) = g(h(x)))*. We will call h a topological weakly br-conjugacy.

Definition 2.10

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:

(1) br-homeomorphism [13] if f is bijective b-irresolute and f^{-1} is also b-irresolute

(2) b-homeomorphism [13] if f is bijective b-continuous and f^{-1} is also b-continuous

Definition 2.11

Two maps $f : X \to X$ *and* $g : Y \to Y$ are said to be topologically br-conjugate if there is a br-homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$. We will call h a topological br-conjugacy.

Remark 2.12

If $\{x_0, x_1, x_2, . . .\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{y_0 = h(x_0), \; y_1 = h(x_1), \; y_2 = h(x_2), . . \}$ yields an orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$. In particular, h maps periodic orbits of f onto periodic orbits of g.

Proposition 2.13

if $f : X \to X$ *and* $g : Y \to Y$ are two maps defined on topological spaces X and Y respectively, suppose that f and g are topologically br-conjugate . Then

(1) f is topologically b- transitive if and only if g is topologically b-transitive;

(2) f is b-minimal if and only if g is b-minimal;

(3) f is topologically b-mixing if and only if g is topologically b-mixing.

Theorem 2.14[3]

For a function $f: X \to Y$ the following properties are equivalent:

1. f is weakly b-irresolute
2. for each element x∈ X and each V∈ BO(Y, f(x)) there exists $U \in BO(X,x)$ such that $f(bCl(U) \subset bCl(V)$
3. $f^{-1}(F) \in BR(X)$ *for every* $F \in BR(Y)$

So, (f is weakly b-irresolute map) ⇔ $f^{-1}(F) \in BR(X)$ *for every* $F \in BR(Y)$ and we will introduce and define topological weakly b-transitive maps as follows:

Definition 2.15

Let (X, τ) be a topological space , $f: X \to X$ be weakly b-irresolute map then f is said to be topological weakly b-transitive if for every pair of non-empty b-regular-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$.

Definition 2.16

(b-minimal) Let X be a topological space and f an b-irresolute map on X. Then (X,f) is called b-minimal system (or f is called b-minimal map on X) if one of the three equivalent conditions hold:

1) The orbit of each point of X is b-dense in X.

2) $Cl_b(O_f(x)) = X$ for each x ϵ X .

3) Given x ϵ X and a nonempty b-open U in X, there exists n ϵ N such that $f^n(x) \in U$.

Theorem 2.17

For (X, f) the following statements are equivalent:

(1) f is b--minimal map.

(2) If E is an b-closed subset of X with $f(E) \subset E$, we say E is invariant. Then E= ϕ or E=X.

(3) If U is a nonempty b-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

Proof:

(1) $\Rightarrow$(2): If A ≠ ϕ , let x ϵ A. Since A is invariant and b-closed, $Cl_b(O_f(x)) \subset A$. On other hand $Cl_b(O_f(x)) = X$. Therefore, $X \subseteq A$ So A=X

(2)$\Rightarrow$(3) Let A=X\ $\bigcup_{n=0}^{\infty} f^{-n}(U)$. Since U is nonempty, A ≠X. Since U is b-open and f is b-irresolute, A is b-closed. Also $f(A) \subset A$, so A must be an empty set.

(3) $\Rightarrow$(1): Let x ϵ X and U be a nonempty b-open subset of X. Since x ϵ X = $\bigcup_{n=0}^{\infty} f^{-n}(U)$ .Therefore x ϵ $f^{-n}(U)$ for some n>0. So $f^n(x) \in U$

Theorem 2.18:

[14] Let (X, τ) be a topological space and $f: X \to X$ be b-irresolute function. Then the following statements are equivalent:

(1) f is b-transitive function

(2) For every nonempty b-open set U in X, $\bigcup_{n=0}^{\infty} f^n(U)$ is b-dense in X

(3) For every nonempty b-open set U in X, $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is b-dense in X

(4) If $B \subset X$ is b-closed and B is f- invariant i.e. $f(B) \subset B$. then B=X or B is nowhere b-dense

(5) If U is b-open and $f^{-1}(U) \subset U$ then U= ϕ or U is b-dense in X.

In [14] we proved that every topologically b-transitive map is transitive map. In this paper every b-transitive map implies weakly b-transitive map, therefore every topologically b-transitive implies both topologically transitive map and weakly b-transitive map. Also we proved that every b-minimal map is minimal map. In this paper every b-minimal map implies weakly b-minimal map, therefore every b-minimal map implies both minimal map and weakly b-minimal map.

3. Conclusion

There are the main results of the paper.

Definition 3.1

A dynamical system (X, f) is called *topologically weakly b-mixing if* for any pair U, V of *non-empty* b-regular open sets there exists $N \in$ N such that for all n ≥ N we have $f^n(U) \cap V \neq \phi$.

Proposition 3.2

Every b-transitive map is a weakly b- transitive map, but the converse not necessarily true, unless, the space X is b-regular space.

Proposition 3.3

Every b- minimal map is a weakly b- minimal map, but the converse not necessarily true.

Theorem 3.4

For (X, f) the following statements are equivalent:

(1) f is an b-minimal map.

(2) If E is an b-closed subset of X with $f(E) \subset E,$ we say E is invariant. Then E=ϕ or E=X.

(3) If U is a nonempty b-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

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